Electromagnet Design Basics for Cold Atom Experiments

Todd P. Meyrath

Atom Optics Laboratory
Center for Nonlinear Dynamics
University of Texas at Austin

November 15, 2001
updated August 18, 2004

1 Introduction

The purpose of the document is to provide basic equations and ideas for fast design of magnetic coils for use as magneto-optical traps, magnetic traps, and field adjustment coils. It was originally written for the author's own reference while designing a QUIC trap for the Raizen Lab's rubidium BEC experiment. Please send comments, questions, and corrections to *meyrath@physics.utexas.edu*.

2 Some Theory of Electromagnets

In this section, I is written for the current in Amp-turns. That is, $I = NI_0$ where N is the number of turns in the electromagnet and I_0 is the actual current in Amps.

2.1 Exact Field of a Circular Current Loop

The field for a circular current loop of current I with radius R displaced from the origin by a distance D as shown in Figure 1 has magnetic field components given by [1]:

$$\begin{aligned} B_z &= \frac{\mu I}{2\pi} \frac{1}{\sqrt{(R+\rho)^2+(z-D)^2}} \left[K(k^2) + \frac{R^2-\rho^2-(z-D)^2}{(R-\rho)^2+(z-D)^2} E(k^2) \right], \\ B_\rho &= \frac{\mu I}{2\pi} \frac{1}{\rho} \frac{z-D}{\sqrt{(R+\rho)^2+(z-D)^2}} \left[-K(k^2) + \frac{R^2+\rho^2-(z-D)^2}{(R-\rho)^2+(z-D)^2} E(k^2) \right], \end{aligned} \tag{1}$$

where

$$k^2 = \frac{4R\rho}{(R+\rho)^2+(z-D)^2}, \tag{2}$$

and $K(k^2)$ and $E(k^2)$ are the complete elliptic integrals for the first and second kind respectively [2].

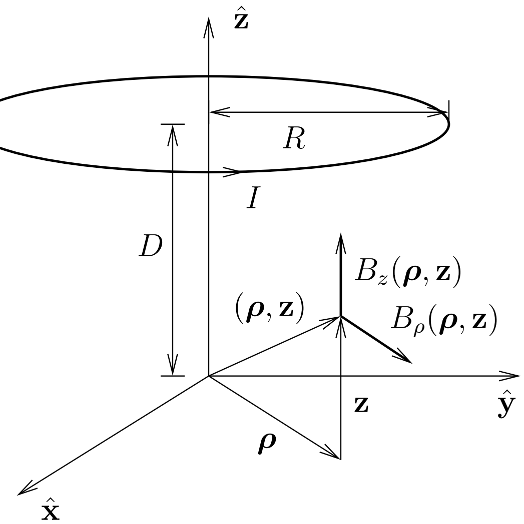

Figure 1: Cylindrical coordinates for the magnetic field of a single circular current loop centered at axial position z.

2.2 Approximations for Circular Current Loops

Figure 1 shows a current loop displaced from the origin, here and in the next sections, we look for approximate field equations near the origin. This corresponding to the case where the coils are somewhere outside a vacuum chamber and all the action is near the origin.

Near the origin ($\rho = 0, x = 0$), we can write a power series expansion for the field components to second order:

$$\begin{aligned} B_z &= \mu I \frac{1}{2} \frac{R^2}{(D^2+R^2)^{3/2}} + \overbrace{\mu I \frac{3}{2} \frac{DR^2}{(D^2+R^2)^{5/2}}}^{\text{axial field gradient}} z \\ &\quad + \underbrace{\mu I \frac{3}{4} \frac{R^2(4D^2-R^2)}{(D^2+R^2)^{7/2}}}_{\text{axial curvature}} (z^2 - \rho^2/2) + \dots , \\ B_\rho &= -\underbrace{\mu I \frac{3}{4} \frac{DR^2}{(D^2+R^2)^{5/2}}}_{\text{radial field gradient}} \rho - \mu I \frac{3}{4} \frac{R^2(4D^2-R^2)}{(D^2+R^2)^{7/2}} z\rho + \dots . \end{aligned} \tag{3}$$

These equations may be used for estimations of fields produced by a single coil. For example, earth field biasing without a Helmholtz pair, one may estimate the single coil field and the deviations from constant field over the region of interest.

2.3 Coil pairs

For actual Helmholtz and anti-Helmholtz pairs we have equations for fields near the origin as follows. The distance between the coils is $d = 2D$.

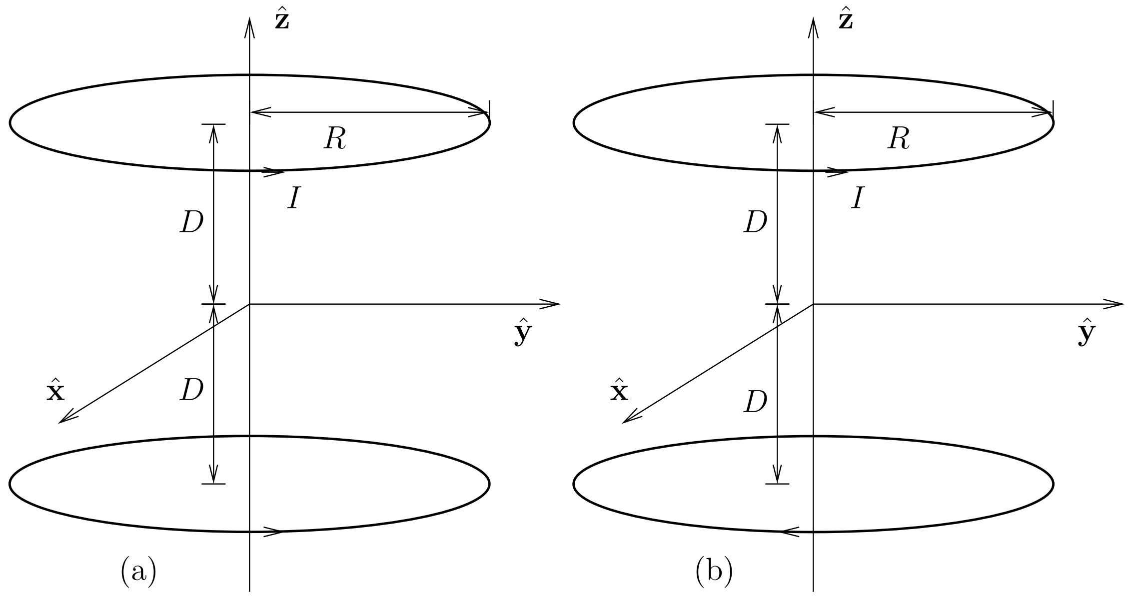

Figure 2: (a) coil pair in Helmholtz configuration. For most ideal constant field, the configuration needs $R = 2D$, *i.e.* the distance between the coils is the same as coil radius. (b) anti-Helmholtz configuration.

In Figure 2(a), a Helmholtz coil configuration is shown, the equations for the field near the origin to third order are:

$$\begin{aligned} B_z &= \mu I \frac{R^2}{(D^2+R^2)^{3/2}} + \mu I \frac{3}{2}\frac{R^2(4D^2-R^2)}{(D^2+R^2)^{7/2}}\,(z^2-\rho^2/2) + \dots\ , \\ B_\rho &= -\mu I \frac{3}{2}\frac{R^2(4D^2-R^2)}{(D^2+R^2)^{7/2}}\, z\rho + \dots\ . \end{aligned} \tag{4}$$

There are no third order terms in for these fields, the next terms are fourth order. In the ideal case with $R = 2D$, the second order terms vanish giving

$$\begin{aligned} B_z &= \mu I \frac{8}{5\sqrt{5}R} + \dots\ , \\ B_\rho &= 0 + \dots\ . \end{aligned} \tag{5}$$

to third order. Naturally, one could simply use the zeroth order term in Equation (4) as a field estimation in case of an imperfect Helmholtz pair which may frequently be the case, such as in earth field nulling coils. For the anti-Helmholtz case shown in Figure 2(b), the equations for the field to third order are:

$$\begin{aligned} B_z &= \mu I 3\frac{DR^2}{(D^2+R^2)^{5/2}}\, z + \mu I \frac{15}{24}\frac{R^2(4D^2-3R^2)}{(D^2+R^2)^{9/2}}\,(4z^3-6\rho^2 z) + \dots\ , \\ B_\rho &= -\mu I \frac{3}{2}\frac{DR^2}{(D^2+R^2)^{5/2}}\,\rho + \mu I \frac{15}{16}\frac{R^2(4D^2-3R^2)}{(D^2+R^2)^{9/2}}\,(\rho^3-4\rho z^2) + \dots\ . \end{aligned} \tag{6}$$

Most notably, the gradient (first order term) in the axial and radial directions differ only by a factor of 2. It is also noted that the third order terms in both directions vanish for $R = \sqrt{4/3}D$, however this is generally not as important as in the Helmholtz case. A more important figure is that the gradient is maximized for $2D = R$, which gives a gradient

$$\frac{dB_z}{dz} = \mu I \frac{48}{25\sqrt{5}R^2} = 2\frac{dB_\rho}{d\rho}. \tag{7}$$

This is to say it is sensible in both cases of Helmholz or anti-Helmholz configuration to space the coils by the coil radius, $spacing = 2D = R = radius$. For most purposes the first order term may be used to estimate the gradient of practical coils in this configuration:

$$B_z \cong \mu I 3 \frac{DR^2}{(D^2+R^2)^{5/2}}\, z, \tag{8}$$

$$B_\rho \cong -\mu I \frac{3}{2} \frac{DR^2}{(D^2+R^2)^{5/2}}\, \rho. \tag{9}$$

3 Numerical Calculations of Fields

It is relatively easy to do a brute-force calculation of the magnetic fields of a more realistic configuration using something like MATLAB. The basic problem with the equations given in the previous section is that they are for infinitesimally thin wires. When one is interested in the fields far from the coil, such a thickness is not relevant, such as when designing typical MOT coils or earth bias coils. However, in the case of a magnetic trap, where the coils may be very close to the region of interest and careful field compensation is required, the size and shape of the coils can make a huge difference. In this situation, it is more critical to calculate the fields carefully for coil sizes as realistic as possible. MATLAB code for exact calculation of the field due to a single circular loop is given in the Appendix. In the function, B_field_loop, one specifies the size, location, and orientation of the loop, and the field is calculated at a point. To do a brute-force calculation for an arbitrarily thick circular coil, one can break that coil into N^2 loops of radius R_n at position r_n^0 and sum the results for each coil. We actually did this for our QUIC trap. For details on this, see the document [3].

4 Power and Cooling

When designing magnetic coils, one important consideration is how to keep them cool. In some instances, when the input power is low (under a few watts) no cooling is needed, tens of watts may be air cooled or water cooled. It really depends on the coil holder structure (metal or plastic) and airflow available. There are other concerns with air-cooling coils in optics experiments, i.e. dust and air index variations. Larger power loads (more than 50 W) are generally water cooled. One comment on heating, is that what I am really discussing here is for rms joule heating. Coils that are pulsed infrequently for short periods may not need to be cooled.

4.1 Power into a Coil

The field of a coil or set at some point in space is

$$B = CI_0N, \tag{10}$$

where C is a constant determined by geometry, I_0 is the current delivered to the coil, and N is the number of turns. Consider this the coil shown in Figure 3. Suppose the coil consists of a length l of magnet wire of some gauge with cross sectional area A_0 and the coil has bulk cross sectional area $A = \pi a^2 = NA_0$. The length is approximately $l \cong 2\pi\bar{R}N$, where $\bar{R}$ is the average coil radius (the radius to the center of the bulk). For copper wire with resistivity, ρ, the net resistance of the coil is $R = \rho l/A_0 \propto N/A_0$. Therefore the power delivered to the coil is $P = I_0^2R \propto 1/NA_0 = 1/A$. Which is to say, for a given desired magnetic field, the power required to be delivered to the coil depends only on the cross sectional area of the coil bulk. So, in order to reduce the amount of joule heating in the coil and produce the same magnetic field, it is necessary to make the coil as thick as possible. This, of course, neglects the changes in field due to geometry changes of thinner or thicker coils. For a coil with given cross section, changing size of wire switches the power needed, $P = IV$, between current and voltage. More useful than this philosophy is the approximate power:

$$P = \frac{2\pi\bar{R}\rho(NI_0)^2}{NA_0}, \tag{11}$$

the resistivity of copper at room temperature is $\rho \cong 1.70 \times 10^{-8}\,\Omega \cdot \mathrm{m}$, $I = NI_0$, of course, is the the number of amp-turns, and $A = NA_0$ is the total coil cross sectional area. If tossing in the area, A, which is to contain the wires, one must, of course, consider the packing fraction, that is, the area of copper is $A \to \alpha\pi a^2$, where α is the packing fraction, and a is the coil cross sectional radius.

4.2 Water Cooling

Water has heat capacity $C = 4186\,\mathrm{J/kg\,^oC}$ and a mass density $\rho = 1.0\,\mathrm{kg/l}$. The change in temperature of water flowing with rate f (in l/s) sinking power P is

$$\Delta T = \frac{P}{\rho C f}. \tag{12}$$

This equation gives the change in temperature from the incoming to the outgoing water which of course depends on the power. The load itself (coil) and the water may have a temperature difference between them. The better the thermal contact between the water and the load the smaller this difference. In the case of our QUIC trap, with flow rate of about $6\,\mathrm{l/min} = 0.1\,\mathrm{l/a}$ and total power of nearly $900\,\mathrm{W}$ the change in temperature is only about $2^o\mathrm{C}$. Since the water to wire contact is so outstanding the wire heats only these few degrees. This temperature change is not a source of instability since it heats the same each cycle. Another comment, in general, is that more surface area of wire contact with water improves heat transfer. This lends something to the philosophy of using smaller diameter and more of it to obtain the same coil cross section.

4.3 Stability of a Magnetic Trap

The stability of magnetic fields is extremely important in a magnetic trap that is used to produce a BEC. The stability depends on three things: current fluctuations, mechanical movements, external field fluctuations. The latter, we don't discuss here, see your local μ-metal distributor. We have not used μ-metal shielding for our magnetic trap. Current fluctuations are due to changes in current supplied to the coil over time. This can be very well controlled as discussed in the document [4]. An important source of fluctuations in magnetic field are due to thermal effects in the coils. This is due to a small amount of thermal expansion of the coil itself on the tens of microns scale for degree changes in temperature which may amount to shifts in field minimum, B_0, of order 1 mG or more. This is important for a BEC but irrelivant for a MOT or other less sensitive application. For our magnetic trap, to get best stability we found it necessary to make sure that the incoming cooling water was stable to under 0.1^oC. This has given us very solid BECs, see [3].

5 Some Standard American Wire Types

Power requirements were discussed in the previous section in general, but ultimately, one must choose a specific wire type for the coils. The table gives typical wire sizes and specifications for standard American wire types. The diameter given is for the bare wire. Insulation, required for any magnet wire, may add between 0.05 and 0.2 mm to the diameter. The given values for resistivity are for copper at 20^{o}C and

Gauge	Diameter mm (in)	Coated † Diam. mm (in)	Possible †† Current A	Line Resistance (ρ/A_0) Ω/km
8	3.251 (0.128)	3.353 (0.132)	50	2.060
10	2.591 (0.102)	2.642 (0.104)	30	3.278
12	2.057 (0.081)	2.108 (0.083)	25	5.210
14	1.626 (0.064)	1.702 (0.067)	20	8.284
16	1.295 (0.051)	1.346 (0.053)	10	13.18
18	1.016 (0.040)	1.067 (0.042)	5	20.95
20	0.813 (0.032)	0.864 (0.034)	3.2	33.30
22	0.635 (0.025)	0.686 (0.027)	2.0	52.95
24	0.508 (0.020)	0.559 (0.022)	1.25	84.22
26	0.406 (0.016)	0.432 (0.017)	0.8	133.9

Table 1: Some Standard American Wire Types [5]. † approximate, depends on the coating type, number of layers, etc. †† depends on cooling, these numbers are for relatively uncooled wires, i.e. they get quite warm.

the currents are a guideline. In general, the allowed current really depends on cooling efficiency. For large coils sinking order 10 W it is relatively unnecessary to cool with water. In a situation with very high current in a very small wire, extremely good water cooling is needed. For example, in our QUIC trap, the Ioffe coil uses 22 gauge wire and is routinely operated at about 30 A with around 400 W to this tiny coil. During early testing of the design, it had been operated at up to 50 A (about 1 kW), but this is no problem for the wire since contact with the flowing water is so good. See the document [3].

6 Inductance and Switching

6.1 Inductance and Parasitics

All coils have inductance, this is the principle limit for switch off time. For a coil of radius $\bar{R}$, thickness $2a$, and number of turns N, shown in Figure 3 the inductance may be approximated by [6]

$$L \cong N^2 \bar{R} \mu \left[\ln \left(\frac{8\bar{R}}{a} \right) - 2 \right]. \tag{13}$$

This equation assumes $\bar{R} \gg a$, which is frequently not the case, but is adequate to estimate for most practical purposes. Figure 4 shows the an equivalent circuit

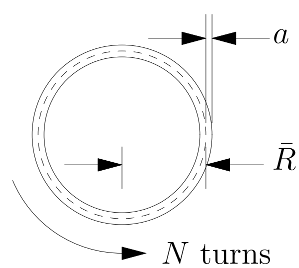

Figure 3: Inductor coil geometry for Equation (13).

for a practical inductor. The resistance is obviously due to the line resistance of the wire used. The capacitance is between the closely spaced turns and depends highly on the wire thickness used, spacing, etc. We have found that the capacitance is typically of order 100 pF down to 1 pF for coils that we have used. These may typically give resonance frequencies in the range of 500 kHz to 10 MHz depending on the coil inductance. For our QUIC trap, the quadrupole coils have $R = 0.29\,\Omega$ (measured), $L \cong 1\,\text{mH}$ (estimated), a resonance frequency $f = 720\,\text{kHz}$ (measured), and a capcitance $C = 1/4\pi^2 L f^2 \cong 50\,\text{pF}$. The Ioffe coil has $R = 0.40\,\Omega$ (measured), $L \cong 0.1\,\text{mH}$ (estimated), a resonance frequency $f = 5.4\,\text{MHz}$ (measured), and a capcitance $C = 1/4\pi^2 L f^2 \cong 10\,\text{pF}$. In general, the important numbers are R and L for power estimations and switch off time. The resonance frequency of the coils is its principle natural frequency and is only really important if, when using a magnetic trap, it is found to resonate with evaporative cooling RF or produce its own RF. This may depend on the circuit using to drive the coils, but interesting to keep in mind for

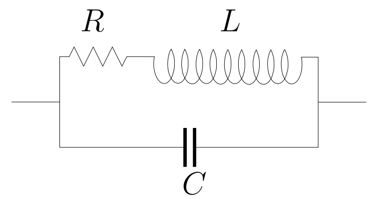

Figure 4: Inductor coil equivalent circuit.

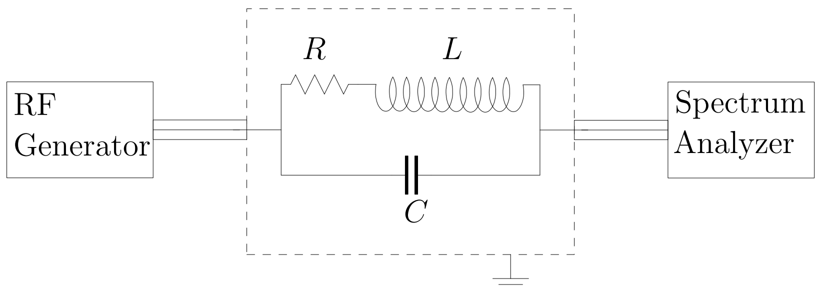

Figure 5: How to measure resonance frequency of a practical coil [7]

system debugging. The resonance frequency may be measured as follows [7]. Pass an RF signal through the coil as shown in Figure 5 to a spectrum analyzer. The signal detected will drop in height at the resonance (to virtually nothing). More practically, the coil will exhibit many resonances, most much weaker.

6.2 Switching

The switching time is mostly limited by the inductance of the coil. A standard switching scheme used for virtually all of our coils is shown in Figure 6. The basic idea is that when the current is suddenly disconnected the voltage across the inductor is clamped at a constant value until all the current is gone. Sudden disconnections are typically done by a Power MOSFET of IGBT, in either case having a zener clamp. In any case, we have a clamping voltage V_{so} (switch off voltage) and an inductance L, this gives the equation $V_{\mathrm{so}} = -L dI/dt$ so the current in time is

$$I(t) = I_0(1 - t/\tau), \tag{14}$$

where the switch off time is

$$\tau = L I_0 / V_{\mathrm{so}}. \tag{15}$$

So it is true that, as inductance goes up switch off time goes up. But it is not as bad as it appears, although $L \propto N^2$, $I_0 \propto 1/N$, so in fact $\tau \propto N$ not N^2. That is, for a coil with a large number of turns, although it takes longer to switch, there is less current to switch, for a given field.

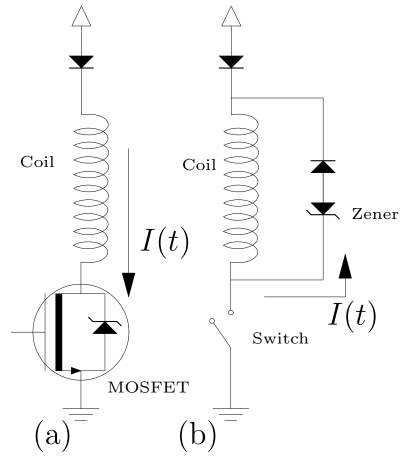

Figure 6: Switching concept circuits. In (a), V_{so} is the reverse breakdown voltage of the MOSFET, typically 100 V to 500 V. Most modern power MOSFETs are internally connected with the zener as shown. IGBTs frequently are not, whereas one connects an external zener diode which may typically more than 1 kV depending on the IGBT. The concept circuit in (b) is basically equivalent to that in (a) with V_{so} as the zener diode voltage.

For our QUIC trap, with an inductance of about 2.1 mH and switch off voltage of 500 V, switching time for a typical current of 28 A is about 120 μs.

6.3 Comment on Steel Chambers

Most steel chambers have μ very close to the free space μ_0, so the chamber itself does not cause a major change in the coil inductance. However, the presence of complete rings with finite resistance will result in eddy currents in the metal when the field is switched off. This causes the eddy currents to support the magnetic field after the coil currents have dropped which means that the fields turn off more slowly. In the case of our QUIC trap, we use a glass chamber between the coils, which gives, very well, the above switching time. In general, this can be a problem if coils are outside a steel chamber. If mounting hardware is made of metal, it can be slotted to reduce this effect. In some cases, plastic hardware can do the job, as in our QUIC trap.

7 Some Philosophy

Most MOT or bias coils are made with standard wires gauges 16 to 24. This is easy and straight forward. In designing a magnetic trap, things are more sensitive. It seems two different approaches are used: (1) use thick wires, few turns, high currents, (2) use thin wires, many turns, low currents. In (1), the thick 'wires' are frequently copper tubes with very small (1/8 inch) inner diameter. This method of cooling has the advantage of good water contact to the (inside of) the conductors. But has the disadvantage of being difficult to work with and requiring booster pumps to get reasonable amount of flow (a few liters per minute for 300 to 400 psi pressures). Booster pumps are noisy and make vibrations. These traps may frequently also require several kilowatts of power, may have order 10 to 50 turns of wire, and take up to 500 A of current. In (2), such as our QUIC trap, thin wire is used in far more turns (150 to 200), lower current (20 to 50 A), powers from 600 W to over a kilowatt. The principle advantage of (1) is the fewer number of turns – the switch off time is shorter. The principle advantage of (2) is likely stability since lower currents are easier to control, and lower powers are easier to tame.

8 Acknowledgments

The author would like to thank his co-workers on the Raizen Lab's rubidium BEC experiment: Florian Schreck, Jay Hanssen, and Chih-sung Chuu. I would also like to thank Kirk Madison for discussions about magnetic traps and of course Prof. Mark Raizen.

Appendix

A Matlab Code for Field Simulation

```
function B = B_field_loop(n,r,r0,R);

%returns the magnetic field from an arbitrary current loop calculated from
%  eqns (1) and (2) in Phys Rev A Vol. 35, N 4, pp. 1535-1546; 1987.
%
%arguments:
% * r is a position vector where the Bfield is evaluated: [x y z]
%     r is in units of d
% * n is normal vector to the plane of the loop at the center, current
%     is oriented by the right-hand-rule.
% * r0 is the location of the center of the loop in units of d
% * R is the radius of the loop
%
%return:
% * B is a vector for the B field at point r in inverse units of
%       (mu I) / (2 pi d)
%    for I in amps and d in meters and mu = 4 pi * 10^-7 we get Tesla
%
% Todd Meyrath
% Atom Optics Lab, University of Texas
% Dec 2001
%

n = n / sqrt( sum(n.*n) );     %normalize n

%choose two vectors perpendicular to n
% choice is arbitrary since the coil is symetric about n
if (abs(n(1))==1)
  l = [n(3) 0 -n(1)];
else
  l = [0 n(3) -n(2)];
end
l = l / sqrt( sum(l.*l) );     %normalize l
m = cross(n,l);
Trans = [l' m' n'];       % transformation matrix coil frame to lab frame
InvTrans = inv(Trans);     % transformation matrix to lab frame to coil frame

r1 = r-r0;   %point location from center of coil
r2 = InvTrans * r1';   %transform vector to coil frame

%%%%calculate field
x = r2(1);
y = r2(2);
z = r2(3);

rho = sqrt( x^2 + y^2 );
if ( (rho==0) & (z==0) )
  Bz = 0;
  Brho = 0;
  co = 0;
  si = 0;
else
  k = sqrt( (4 * R * rho)/( (R + rho)^2 + z^2) );
  [K,E] = ellipke(k^2);
  Bz = ( 1/sqrt((R + rho)^2 + z^2) )*( K + E*(R^2-rho^2-z^2)/((R-rho)^2+z^2) );
  if (rho==0)
    Brho = 0;
    co = 0;
    si = 0;
  else
    Brho = ( z/(rho*sqrt((R + rho)^2 + z^2)) )*( -K + E*(R^2+rho^2+z^2)/((R-rho)^2+z^2) );
```

```
    co=(x/sqrt(x^2+y^2));
    si=(y/sqrt(x^2+y^2));
  end
end

B(1)=co*Brho;
B(2)=si*Brho;
B(3) = Bz;

B = Trans * B';
```

References

[1] Metcalf Bergeman, Erez. Magnetostatic trapping fields for neutral atoms. *Phys. Rev. A*, 35:1535–1546, 1987.

[2] Liu Spiegel. *Mathematical Handbook of formulas and tables*. McGraw Hill, 2rd edition, 1999.

[3] Todd Meyrath. Design of a quadrupole ioffe configuration magnetic trap. 2003.

[4] Todd Meyrath. Current controller designs for cold atom experiments. 2003.

[5] Coplan Moore, Davis. *Building Scientific Apparatus*. Perseus Books, 2nd edition, 1991.

[6] Van Duzer Ramo, Whinnery. *Fields and Waves in Communication Electronics*. Wiley, 3rd edition, 1994.

[7] J. Carr. *Secrets of RF Circuit Design*. McGraw Hill, 2001.